



AN015: Aperture and F/#

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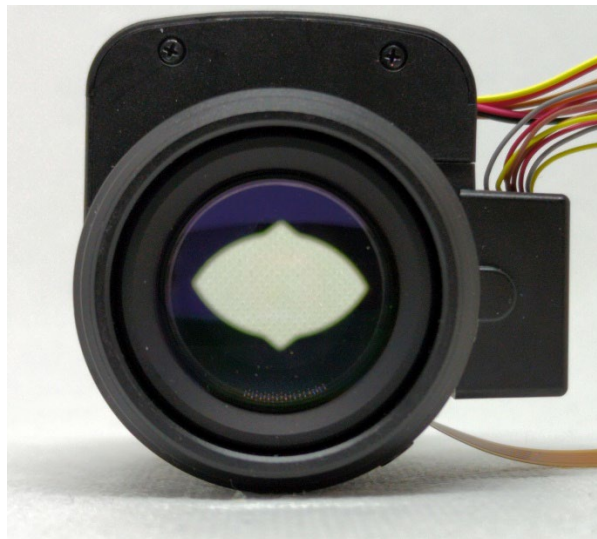
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1. Aperture and F/#

This calibration data frame is applicable to lenses with a p-iris.

The aperture used in Theia's p-iris lenses is not round so calculation of F/# and numeric aperture is based on the area of the iris and is then converted to numeric aperture. This numeric aperture is used for control points in other data frames such as image resolution.

The orientation of the axis in the lens is fixed so the short and long axes of the iris will be consistent from lens to lens. The orientation is shown in figures 1.1 and 1.2.



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Fig 1.1: aperture orientation in Theia's TL1250 lenses



Fig. 1.2: aperture orientation in Theia's TL410 lenses

A typical use, discussed in this application note, is to set the lens iris motor step based on the desired F/# or depth of field.

2. Read the data file

The calibration curve data is at the top level of the JSON data in the data file, found under two keys.

2.1. Data file key "AP"

The key "AP" is data for the calibration curve that relates the numeric aperture (calculated per the method shown below) to the iris motor step number and can be used for any lens light gathering calculations. "AP" is based on the area of the iris opening. Focal length will affect the values and is used as the one control point.

2.2. Data file key "iris"

The key "iris" relates the short diameter of the iris to the iris step and is used for depth of field calculations. This iris short diameter is a component of the numeric aperture calculation but is provided as an accessible parameter. There is no need to extract the short diameter from the calculated numeric aperture. This parameter can be used in the calculation of depth of field.

2.3. JSON data format

This data is measured on a population sample of lenses but it is not measured for each lens individually. It can be considered more similar to "design" data than to "calibrated" data and the "type" key indicates this.

In this example for Theia's TL1250P N6 lens, there is only 1 calibrated curve at 12.5mm focal length. It is possible to have additional curves with different focal length control parameters.

```
{  
  ...  
  "AP": {  
    "type": "des",
```

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```

    "idx": 6,
    "count": 1,
    "xAxis": "iris step",
    "xMin": 0,
    "xMax": 75,
    "yAxis": "Aperture",
    "cp1Type": "FL [mm]",
    "cp2Type": "",
    "cp1": [
        26.6
    ],
    "cp2": [
        0
    ],
    "coef": [
        [
            2.6900000E-01
            2.8900000E-03
            -1.0080833E-03
            6.7020833E-05
            -2.1020833E-06
            3.0791667E-08
            -1.7083333E-10
        ]
    ]
},
"iris": {
    "type": "des",
    "idx": 15,
    "count": 1,
    "xAxis": "iris step",
    "xMin": 0,
    "xMax": 75,
    "yAxis": "Dia (short)",
    "cp1Type": "FL [mm]",
    "cp2Type": "",
    "cp1": [
        26.6
    ],
    "cp2": [
        0
    ],
    "coef": [
        [
            -1.2342607E+01
            -2.05710117E-01
        ]
    ]
}, ...

```

The polynomial coefficients are listed from the constant term ($P_0 \cdot x^0$) up to the 10th order ($P_{10} \cdot x^{10}$).

$$y = \sum_{k=0}^{10} P_k * x^k$$

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They form a calibration curve of aperture (y-axis) vs. iris motor step number (x-axis) as shown.

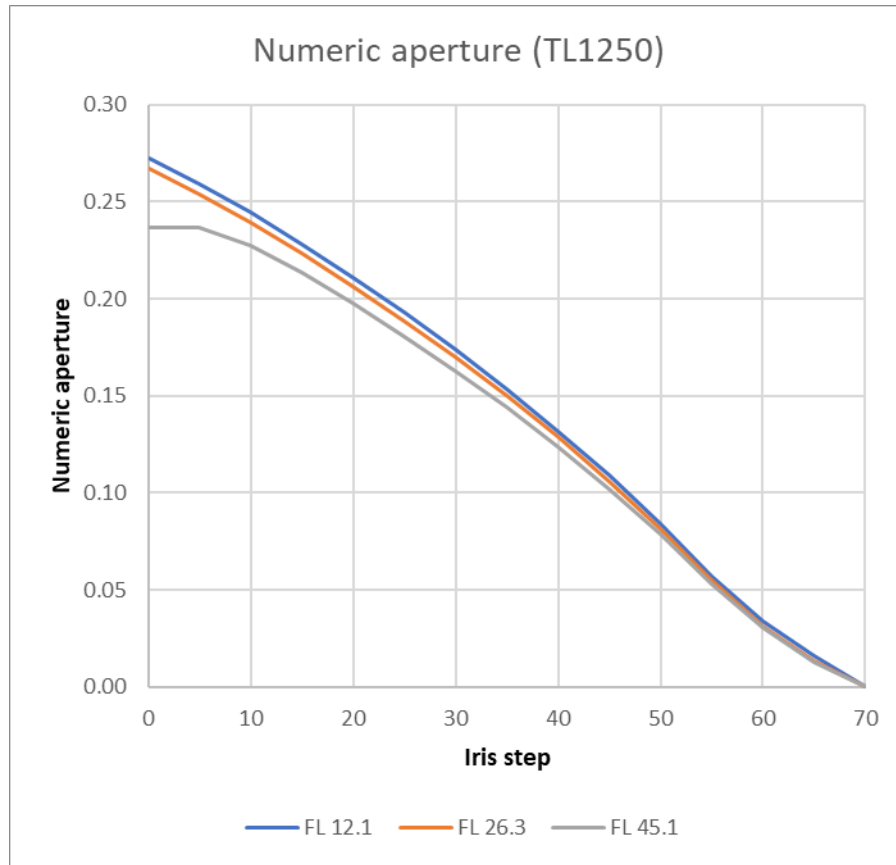


Fig. 2.1. Calibration graph for "AP": numeric aperture (based on aperture area) vs. step

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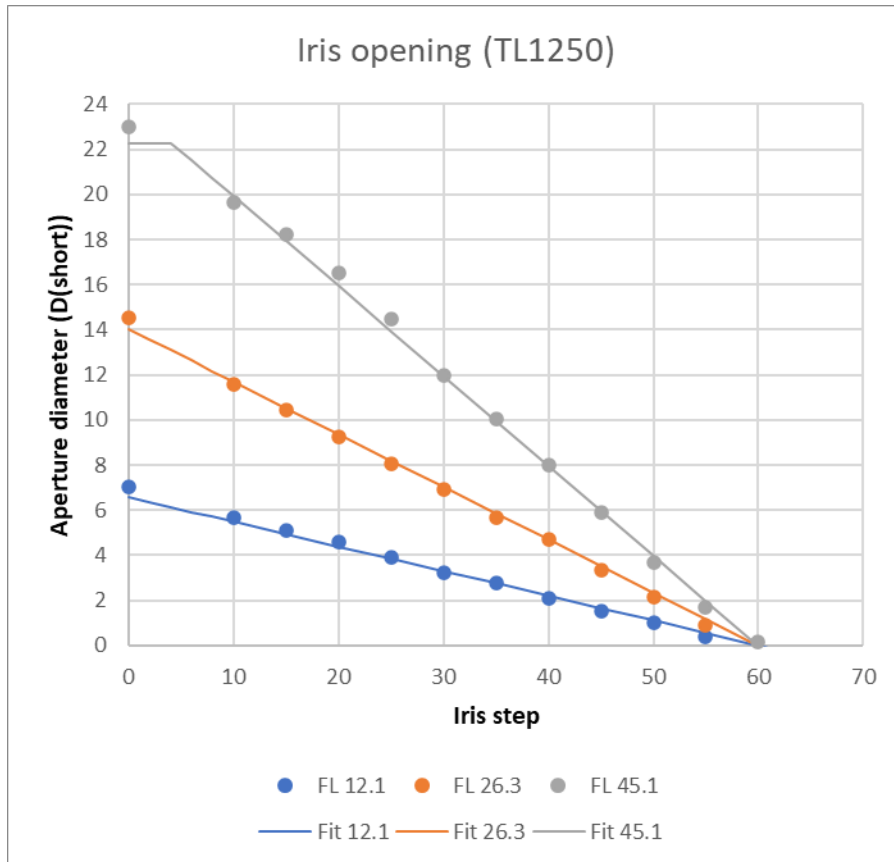


Fig 2.2. Calibration graph for “iris”: short diameter (D_{short}) vs. step. Iris curves are linear.

3. Data tables for Theia lenses

3.1. TL1250 lens

Step	FL 12.1		FL 24		FL 48	
	F/#	Aperture	F/#	Aperture	F/#	Aperture
0	1.84	0.272	1.87	0.267	2.10	0.238
5	1.93	0.259	1.97	0.254	2.10	0.238
10	2.05	0.244	2.09	0.240	2.19	0.228
15	2.19	0.228	2.24	0.224	2.34	0.214
20	2.37	0.211	2.42	0.207	2.53	0.198
25	2.59	0.193	2.65	0.189	2.76	0.181
30	2.88	0.174	2.94	0.170	3.06	0.163
35	3.26	0.153	3.33	0.150	3.46	0.144
40	3.79	0.132	3.87	0.129	4.03	0.124
45	4.59	0.109	4.70	0.106	4.89	0.102
50	5.95	0.084	6.11	0.082	6.35	0.079
55	8.74	0.057	9.08	0.055	9.43	0.053
60	14.72	0.034	15.84	0.032	16.48	0.030
65	4.33	0.116	1.45	0.344	0.43	1.173

4. Aperture size calculation (background)

For photographic lenses in air, aperture is approximated¹ as $na = \frac{1}{2 * F/\#}$ where $F/\# = \frac{FL}{D}$. This works well when the iris is round and has a well-defined diameter. However, in Theia's lenses with p-iris, the iris is made with only 2 leaves. When the iris is fully open, the aperture is a perfect circle as defined by a circular stop. Closing the iris by moving the 2 leaves in changes the iris shape to more oblong.

Figure 3.1 shows the iris closing vertically (iris step 40 of 75 is shown). The leaves have an arc that matches the fully open stop diameter but also includes a small triangular section which makes it easier to control the aperture area when nearly closed. Because of this shape, calculating the aperture is not straightforward.

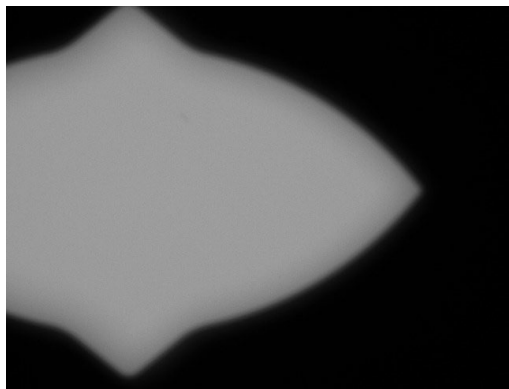


Fig 3.1. Lens aperture at iris step 40 (1/2 closed).

$F/\#$ and aperture are calculated based on the area of the aperture. This will give an average value for other calculations. The area is twice the sum of the partial circle (A_C) and the small triangle (A_T). The triangle is only added if the leaves are partially closed and away from the full-open circular aperture.

Figure 3.2 shows the iris leaf closing (red diagonal pattern) and the triangle portion of the iris leaf (green enclosed area) is almost fully contributing to the area at iris motor step 7 (for TL1250 lens) but the top is just clipped by the circular aperture in the lens.

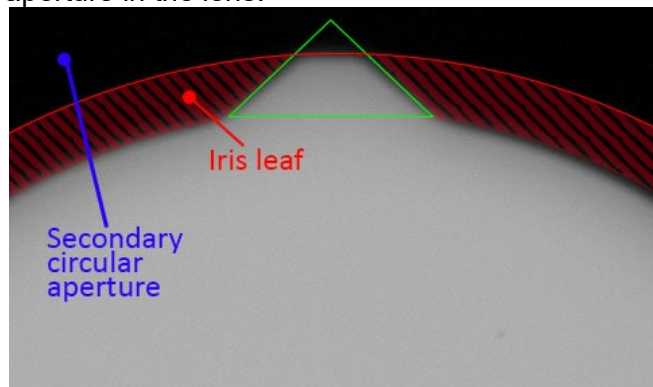


Fig. 3.2. Aperture triangle section clipped by circular aperture until after step 7 (shown).

Calculate the major contribution to the aperture area $A_C = A_F - A_B$ per figure 3.3.

¹ https://en.wikipedia.org/wiki/Numerical_aperture

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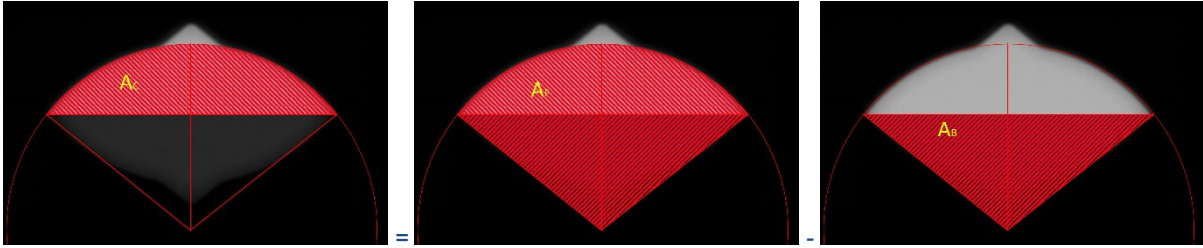


Fig 3.3. Calculate the area A_c

Area of the arc of a circle $A_F = \frac{1}{2}r^2\theta$ where $\frac{\theta}{2} = \cos^{-1}\frac{s}{r}$

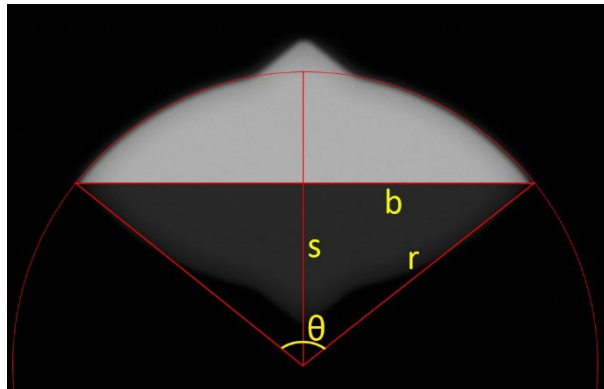


Fig. 3.4. Calculation of the major contributor of the area

Area of the base triangle $A_B = sb$ where $b = \sqrt{r^2 - s^2}$. Therefore

$$A_C = r^2 \cos^{-1} \frac{s}{r} - s\sqrt{r^2 - s^2} \quad \text{Eq 3.1}$$

where s is related to the iris motor step number and is affected by the focal length of the lens. Magnification of the pupil will change the values by a scalar amount.

The small triangular section of the aperture can also be added (figure 3.5). This should only be added if the iris leaves are closed sufficiently to allow light through this portion.

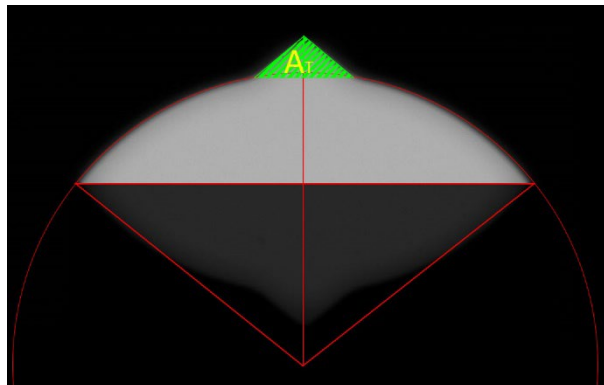


Fig. 3.5. Additional triangular aperture area

The triangle has a 90° angle so the area is $A_T = h^2$. If part of this area is cut off by the limiting circular aperture the contributing area A_s is limited by the radius r and is

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$$A_s = ((r - s) + h) - r$$

If $A_s < 0$ then the small triangle is not cut off by the circular aperture and total area should be included in the calculation. Subtracting A_s from the triangle area results in

$$A_T = h^2 - \max(0, (h - s))^2 \quad \text{Eq 3.2}$$

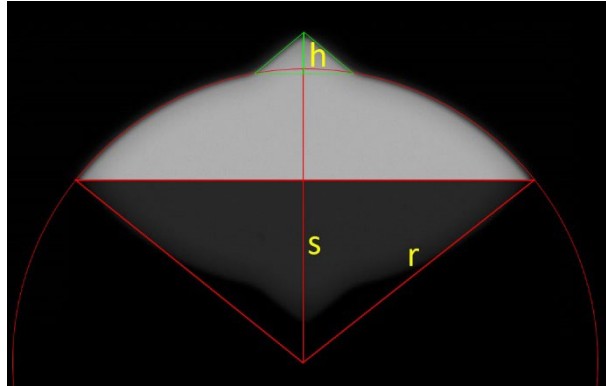


Fig. 3.6. Calculation of the additional triangle area

This should be doubled to account for the entire aperture.

$$A_{total} = 2 * (A_C + A_T) \quad \text{Eq 3.3}$$

which can be used to calculate the “average” F/# and aperture values.

$$F\# = \frac{FL}{2\sqrt{\frac{A_{total}}{\pi}}} \quad \text{Eq 3.4}$$

$$na = \frac{\sqrt{\frac{A_{total}}{\pi}}}{FL} \quad \text{Eq 3.5}$$

5. Depth of field

Depth of field will be slightly different in the two axes of the aperture due to the non-round shape. The difference is small but it can be calculated as shown below.

For the short axis (vertical in the images) diameter of the axis, there is a contribution due to the small triangular section of the aperture which becomes greater as the iris approaches fully closed. However over the majority of the useful aperture settings (any iris step less than 40, aperture faster than F/5) this contribution is small and can be ignored. The diameter can be calculated from the iris motor step number using the “iris” calibration curve.

$$D_{short} = \sum_{k=0}^1 P * n^k$$

Which in this case can be re-written as

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$$D_{short} = P_1 * (n - n_0)$$

Where n is the motor step number and n_0 is the step number where the iris is closed. For Theia lenses, $n_0 = 60$.

Note for Theia's TL1250 lens the iris is the limiting circular aperture D_{max} for most of the focal lengths. However when the focal length becomes greater than ~35mm, there are other apertures in the lens that become the limiting apertures. This causes the F/# to increase at the telephoto end. In the TL1250 case, the maximum aperture is a non-linear polynomial:

$$D_{max} = \sum_{k=0}^3 P * FL^k$$

TL1250 maximum aperture	
P0	2.814642E+00
P1	2.257034E-01
P2	1.332650E-02
P3	-1.944634E-04

For the long axis of the aperture (horizontal in the images), the diameter is the intersection of the two round sections of the aperture leaves. This has already been shown to be $2 * b$ where the value $s = D_{short}$ and $r = D_{short} |_{iris=0}$ (maximum short aperture at iris motor step 0)

$$D_{long} = 2 * \sqrt{r^2 - D_{short}^2}$$

Because of the non-round shape of the aperture, the calculated depth of field will not be exact. As the aperture closes it becomes less circular resulting in an underestimate of the depth of field in the long axis.

A useful calculation is the minimum object distance that is still considered acceptably in focus. This can be calculated² using either the short or long diameter of the aperture. The short diameter is used for this calculation shown below. From the reference equation in the Wikipedia reference:

$$b = dm_s x_d / D$$

by substituting variable names and rearranging the minimum and maximum object distances can be calculated.

$$Z_{ext} = \frac{Z}{1 \pm \left(\frac{CoC}{D_{short} * M} \right)} \quad \text{Eq 4.1}$$

Where Z is the nominal, focused object distance and Z_{ext} is the minimum (+ in denominator) or maximum (- in the denominator) object distance that is acceptably in focus. The circle of least confusion is CoC. The magnification, M , can be approximated for large object distances as:

$$M = \frac{FL}{Z}$$

The CoC has been set historically by film cameras as the blur spot size that is considered “acceptably good focus”. For 35mm film this was set as 0.050mm but has recently been made smaller. It is a subjective number depending on the application and sensor pixel size.

² https://en.wikipedia.org/wiki/Depth_of_field

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Based on empirical experiments using a 6MP 1/1.8" sensor the image sharpness was evaluated at different object distances. For this smaller sensor the circle of confusion can be estimated between 15µm and 20µm.

$$CoC \approx 0.020mm$$

Using Theia's TL1250 telephoto as an example:

CoC = 0.020mm,

FL = 48mm,

best focus object distance = 2520mm,

aperture diameter D_{short} = 24mm

the minimum and maximum in-focus object distances can be calculated:

$$Z_{ext} = \frac{2520}{1 \pm \left(\frac{0.020 * 2520}{24 * 48} \right)} = \frac{2414}{2635} mm = \frac{241}{263} cm$$

This is seen empirically. The "acceptably good focus" range is from 242cm to 262cm object distance. This is a total DOF of about 20cm which corroborates the 20µm CoC.

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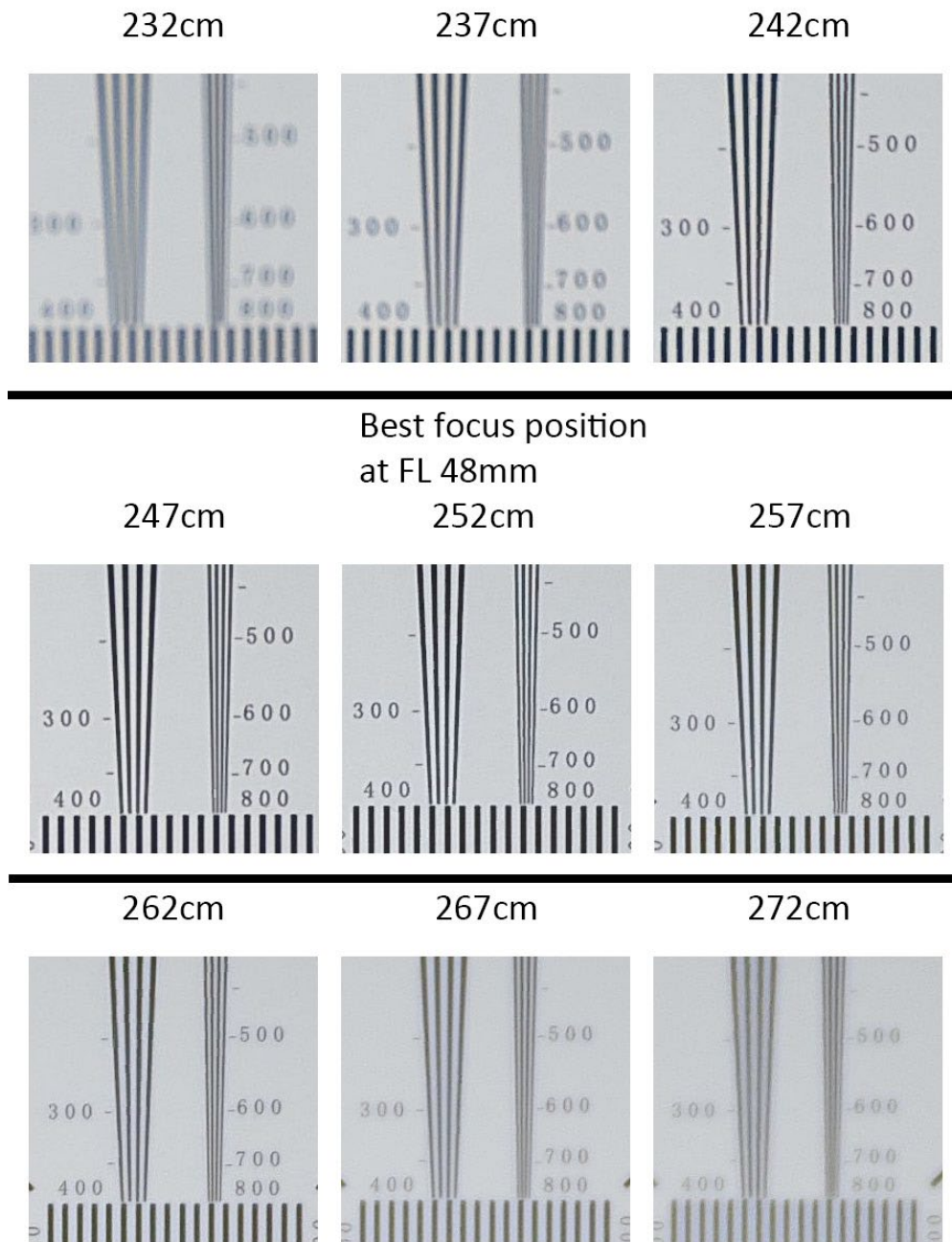


Fig. 4.1. Theia's TL1250 lens at 48mm and different object distances (full open iris)

There are many variables that should be considered when calculating CoC and corresponding DoF including the intended application for the image, image sensor pixel size, imaging optics resolution, and image processing. Also when the aperture is partially closed the iris becomes non-circular which will affect the DoF differently in horizontal and vertical directions.

6. Depth of field calculation (background)

Theia's lenses have 2 leaves to control the aperture opening as shown in section 3. The leaves have a shape that allows fine control of the aperture when it is almost closed by adding a triangular cut-out section. This triangular cutout is small relative to the rest of the aperture opening when the aperture is

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mostly open so it can be ignored when calculating the depth of field. The aperture short diameter doesn't include this triangular section.

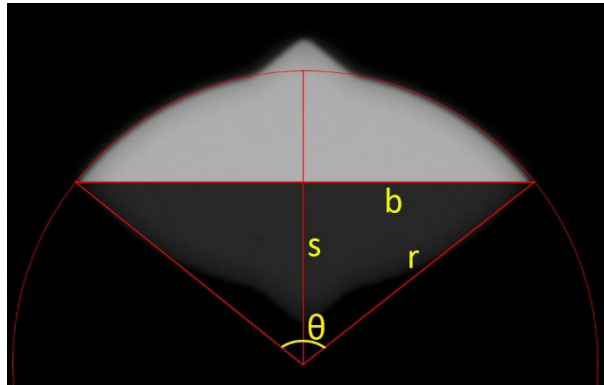


Fig. 5.1. Calculation of the aperture half-diameter

Based on figure 5.1, the short diameter can be calculated.

$$D_{short} = 2 * (r - s)$$

The aperture stepper motor directly drives the aperture leaves. Because of the low total angle of travel of the motor it is very nearly linear. The aperture opening is linearly related to the iris motor step value.

$$D_{short} = (n - n_0) * m|_{FL} \quad \text{Eq. 5.1}$$

Where m is the slope of the relationship and is a function of focal length, n is the motor step number, and n_0 is the point where the aperture circular part is closed. For Theia's lenses (TL936, TL410, and TL1250) this value is $n_0 = 60$. When "closed", there is still some aperture clear area due to the small triangular sections but at these small aperture sizes, the calculation assumptions are starting to break down.

For Theia's TL1250 lens series, because of the position of the aperture inside the lens, the slope of this aperture opening v. step number line is a linear function of the focal length. This simplifies the calculation of the aperture opening for any focal length.

$$m|_{FL} = FL * m_{TL1250} + b_{TL150} \quad \text{Eq 5.2}$$

Where m_{TL1250} and b_{TL1250} are constants related to the lens series.

The aperture full diameter (r) is also a linear function of focal length. This simplifies the calculation. Based on this additional linear relation, the size of the small triangular section, required for calculating the numeric aperture, can easily be scaled from a known, fixed maximum aperture diameter.

$$D_{max} = FL * m_{Dmax} + b_{Dmax} \quad \text{Eq 5.3}$$

7. Calculation examples

Find the aperture and depth of field at focal length 26.3mm and iris step 10:

Fixed values	
Lens	TL1250

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Maximum aperture diameter at maximum focal length 48.5mm	$D_{\max} = 25.4\text{mm}$
Small triangle height	$h = 1.80\text{mm}$
Aperture v. step slope	$m_{\text{TL}1250} = -0.008741706$
Aperture v. step intercept	$b_{\text{TL}1250} = -0.004099199$
Maximum aperture v. focal length slope	$m_{D_{\max}} = 0.503760225$
Maximum aperture v. focal length intercept	$b_{D_{\max}} = 0.965271191$

Calculate the depth of field:

Aperture diameter v. iris step linear equation slope	$m _{26.3} = -0.234$	Eq 5.2
Aperture diameter at iris step 10	$D_{\text{short}} = 11.7\text{mm}$	Eq 5.1
Depth of field at object distance 2.5m	$\text{DoF} = 81\text{cm}$	Eq 4.2

Calculate the numeric aperture the detailed way:

Maximum aperture diameter at 26.3mm	$D_{\max(26.3\text{mm})} = 14.2\text{mm}$	Eq 5.3
$s = (D_{\max} - D_{\text{short}}) / 2$	$s = 1.257$	
Calculate maximum radius $r = D_{\max} / 2$	$r = 7.107$	
Calculate A_C	$A_C = 61.56$	Eq 3.2
Calculate the triangle height h by scaling D_{\max}	$h = 1.007$	
Calculate A_T	$A_T = 1.01$	Eq 3.3
Calculate total area	$A_{\text{total}} = 125.17$	Eq 3.4
Calculate numeric aperture	$\text{na} = 0.240$	Eq 3.5
Calculate F/#	$F/\# = 2.08$	Eq 3.4

8. Revisions

Version	Change	Reason
221122		Preliminary release
230306	Added DoF section	
230404	Added DoF background section	Information on the linearity of aperture vs. motor step
	Added calculation example and data table	Allows implementation confirmation
230725	Depth of field calculation changed to min/max object distance	Depth of field full range is less useful than min acceptable object distance in focus